

# Fault-distribution-dependent reliable control for time-varying delay system

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**Abstract:** This paper considers the problem of reliable control for continuous-time systems with interval time-varying delay. By introducing a random matrix, a new practical actuator fault model is established. Using the Lyapunov-Krasovskii approach, a sufficient condition for the existence of reliable controller is expressed by a linear matrix inequality(LMI). An illustrative example is exploited to show the effectiveness of the proposed design procedures.

**Keywords:** Reliable control; Stochastic actuator fault model; Time-delay

## 1 Introduction

Time-delay phenomenon is often encountered in various practical systems, such as distributed networks, chemical engineering systems, inferred grinding models, microwave oscillator, manual control, neural networks, ship stabilization, population dynamic model, and systems with lossless transmission lines. The existence of the time delay may cause instability or bad performances in dynamic systems. Hence, the stability and stabilization problems for the systems with time-delay have received some attenuation [1–4].

The actuator is an important component of control systems, so its failure may deteriorate the control system performance and may even cause systems instability in practical control systems. Reliable control, introduced to tolerate the failures and to maintain the system stability and performance, is therefore more meaningful. However, most exiting results are based on the assumption that the actuators are in good condition. In fact, actuator signal drift including complete failure often occurs in real world. The main task of this study is to establish a reasonable actuator fault model and design a reliable controller based on this model, such that the closed-loop system can maintain its stability and performance, not only when all control components are operational, but also in the case of existing some abnormal actuators.

Over the past few decades, reliable control problems have been extensively studied [5–11]. Most of these studies depict the failure model by introducing a scaling factor  $\beta_i, \beta_i \in \Omega \triangleq \{\beta_i = \text{diag}\{\beta_{i_1}, \beta_{i_2}, \dots, \beta_{i_q}\}, \beta_{i_i} = 0 \text{ or } 1, i = 1, 2, \dots, q\}$ , that is, the actuator will lost its all functions when the actuator failure occurs, in fact, this was not the case. The scale factor  $\beta_{i_i} = 0$  or  $1$  are only two special cases. Another modelling approach is by decomposing the control matrix  $B$  into  $B_\Sigma$  and  $B_{\bar{\Sigma}}$  [12, 13], where  $B_\Sigma$

denotes the control matrix associated with the set  $\Sigma$  and  $B_{\bar{\Sigma}}$  denotes the control matrix associated with the complementary subset of the control input, and  $B_\sigma$  with  $\sigma \subseteq \Sigma$  correspond to a subset of susceptible actuator experience failure.

However, in most situations, the gain of the actuator fluctuates over disturbance with a certain distribution. The existing actuator fault model will not apply here. In this paper, we replace the fault scale factor with a random variable which obeys a certain probabilistic distribution in an interval. To the best of our knowledge, it seems that there are few results on such an actuator fault model, which is not only theoretically interesting and challenging, but also very important in practical applications. This greatly motivates the present work.

In this paper, a more general actuator fault model is proposed, which satisfies a certain probabilistic distribution in an interval. We are interesting in designing a reliable controller such that the dynamic system is exponentially mean-square stable despite possible actuator signals drift or missing. Then, sufficient conditions for the existence of reliable controller are established in terms of linear matrix inequalities (LMIs). Finally, a numerical example is provided to demonstrate the effectiveness of the proposed design approach.

## 2 Problem formulation

Consider the following continuous-time system with interval time-varying delay:

$$\dot{x}(t) = Ax(t) + A_d x(t - \tau(t)) + Bu(t), \quad (1)$$

$$x(t) = \phi(t), \quad t \in [-\tau_2, -\tau_1], \quad (2)$$

where  $x(t) \in \mathbb{R}^n$  is the state,  $u(t) \in \mathbb{R}^m$  is the control input,  $\phi(t)$  is a continuous vector-valued initial function,  $\tau(t)$  denotes the state delay and satisfies  $\tau_1 \leq \tau(t) \leq \tau_2$ ,  $A, A_d$

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$$\begin{aligned}
 &+ \begin{bmatrix} x(t) \\ x(t - \tau_1) \end{bmatrix}^T \begin{bmatrix} -R_1 & R_1 \\ R_1 & -R_1 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t - \tau_1) \end{bmatrix} \\
 &+ \begin{bmatrix} x(t) \\ x(t - \tau(t)) \\ x(t - \tau_2) \end{bmatrix}^T \begin{bmatrix} -R_2 & R_2 & 0 \\ * & -2R_2 & R_2 \\ * & * & -R_2 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t - \tau(t)) \\ x(t - \tau_2) \end{bmatrix} \\
 &+ \begin{bmatrix} x(t - \tau_1) \\ x(t - \tau(t)) \\ x(t - \tau_2) \end{bmatrix}^T \begin{bmatrix} -R_3 & R_3 & 0 \\ * & -2R_3 & R_3 \\ * & * & -R_3 \end{bmatrix} \begin{bmatrix} x(t - \tau_1) \\ x(t - \tau(t)) \\ x(t - \tau_2) \end{bmatrix}.
 \end{aligned}$$

Hence,

$$\mathcal{L}V(x_t) \leq \mathcal{E}\{\zeta^T(t)\Omega\zeta(t)\}, \tag{9}$$

where  $\zeta(t) = [x^T(t) \ x^T(t - \tau_1) \ x^T(t - \tau(t)) \ x^T(t - \tau_2)]^T$ ,  $\mathcal{R}$  and  $\Omega$  are defined in Theorem 1.

$$\Theta = (\Theta_{ij})_{6 \times 6} = \begin{bmatrix} \tilde{\Gamma}_{11} & \tilde{R}_1 & A_d X + \tilde{R}_2 & 0 & XA^T + Y^T \tilde{E}^T B^T & \tilde{A} \\ * & -\tilde{R}_1 - \tilde{R}_3 - \tilde{Q}_1 & \tilde{R}_3 & 0 & 0 & 0 \\ * & * & -2\tilde{R}_2 - 2\tilde{R}_3 & \tilde{R}_2 + \tilde{R}_3 & XA_d^T & 0 \\ * & * & * & -\tilde{R}_2 - \tilde{R}_3 - \tilde{Q}_2 & 0 & 0 \\ * & * & * & * & -2X + \tilde{\mathcal{R}} & 0 \\ * & * & * & * & * & -2\tilde{X} + \tilde{\mathcal{R}} \end{bmatrix} < 0, \tag{11}$$

where

$$\begin{aligned}
 \tilde{\Gamma}_{11} &= AX + XA^T + B\tilde{E}Y + Y^T \tilde{E}^T B^T \\
 &\quad + \tilde{Q}_1 + \tilde{Q}_2 - \tilde{R}_1 - \tilde{R}_2, \\
 \tilde{A} &= [\sigma_1 Y^T C_1^T B^T \ \sigma_2 Y^T C_2^T B^T \ \dots \ \sigma_m Y^T C_m^T B^T], \\
 \tilde{\mathcal{R}} &= \text{diag}\{\underbrace{\tilde{\mathcal{R}}, \dots, \tilde{\mathcal{R}}}_m\},
 \end{aligned}$$

$$\begin{bmatrix} \Gamma_{11} & R_1 & PA_d + R_2 & 0 & \bar{A}^T P & \bar{\mathcal{A}}\bar{P} \\ * & -R_1 - R_3 - Q_1 & R_3 & 0 & 0 & 0 \\ * & * & -2R_2 - 2R_3 & R_2 + R_3 & A_d^T P & 0 \\ * & * & * & -R_2 - R_3 - Q_2 & 0 & 0 \\ * & * & * & * & -P\mathcal{R}^{-1}P & 0 \\ * & * & * & * & * & -\bar{P}\bar{\mathcal{R}}^{-1}\bar{P} \end{bmatrix} < 0, \tag{12}$$

$$\begin{bmatrix} \Gamma_{11} & R_1 & PA_d + R_2 & 0 & \bar{A}^T P & \bar{\mathcal{A}}\bar{P} \\ * & -R_1 - R_3 - Q_1 & R_3 & 0 & 0 & 0 \\ * & * & -2R_2 - 2R_3 & R_2 + R_3 & A_d^T P & 0 \\ * & * & * & -R_2 - R_3 - Q_2 & 0 & 0 \\ * & * & * & * & -2P + \mathcal{R} & 0 \\ * & * & * & * & * & -2\bar{P} + \bar{\mathcal{R}} \end{bmatrix} < 0, \tag{13}$$

where

$$\begin{aligned}
 \mathcal{A} &= [\sigma_1 K^T C_1^T B^T \ \sigma_2 K^T C_2^T B^T \ \dots \ \sigma_m K^T C_m^T B^T], \\
 \mathcal{R} &= \text{diag}\{\underbrace{\mathcal{R}, \dots, \mathcal{R}}_m\}, \\
 \bar{P} &= \text{diag}\{\underbrace{P, \dots, P}_m\}.
 \end{aligned}$$

Due to

$$(\mathcal{R} - P)R^{-1}(\mathcal{R} - P) \geq 0,$$

which gives

$$-P\mathcal{R}^{-1}P \leq -2P + \mathcal{R}, \tag{14}$$

and we have that (12) holds if (13).

From (6) and (9), we can conclude:

$$\mathcal{L}V(x(t)) < -\lambda_{\min}(\Omega)\mathcal{E}\{\zeta^T(t)\zeta(t)\}, \tag{10}$$

where  $\lambda_{\min}$  is the minimum eigenvalue of  $\Omega$ .

Then, we can conclude system (5) is EMSS by using the similar method of Theorem 1 in [22]. The proof is completed.

In the following, we are seeking to design the reliable controller gain  $K$  based on Theorem 1.

**Theorem 2** For given scalars  $\tau_1, \tau_2, \sigma_i, \mu_i (i \in \mathfrak{M})$ , there exists a static state feedback reliable controller in the form (4) such that closed-loop system (5) is EMSS, if there exist matrices  $X > 0, \tilde{Q}_i > 0 (i = 1, 2), \tilde{R}_j > 0 (j = 1, 2, 3)$  and matrix  $Y$  satisfy LMI (11). Furthermore, the reliable controller gain is  $K = YX^{-1}$ .

$$\tilde{\mathcal{R}} = \tau_1^2 \tilde{R}_1 + \tau_2^2 \tilde{R}_2 + (\tau_2 - \tau_1)^2 \tilde{R}_3,$$

$$\tilde{X} = \text{diag}\{\underbrace{X, \dots, X}_m\}.$$

**Proof** By using Schur complement, equation (6) holds if and only if (12) shown.

Defining  $X = P^{-1}, \bar{X} = \text{diag}\{\underbrace{X, \dots, X}_m\}$  and applying

the congruence transformation  $\text{diag}\{X, X, X, X, X, \bar{X}\}$  to (13) and setting  $\tilde{Q}_i = XQ_iX (i = 1, 2), \tilde{R}_j = XR_jX (j = 1, 2, 3)$  and  $Y = KX$ , we can conclude the result from equation (13). This completes the proof.

**Remark 3** There exists conservatism in step equation (13)  $\Rightarrow$  equation (12) by using equation (14). The results will be a little improved if adopting the cone complementary algorithm [23], which is a popular method in dealing with controller designs. To avoid using algorithms, we can introduce a scaling parameter  $\varepsilon > 0$  to improve the LMI

condition in Theorem 2. That is

$$\begin{aligned} (\mathcal{R} - \varepsilon^{-1}P)R^{-1}(\mathcal{R} - \varepsilon^{-1}P) &\geq 0 \\ \Rightarrow -P\mathcal{R}^{-1}P &\leq -2\varepsilon P + \varepsilon^2\mathcal{R}. \end{aligned} \quad (15)$$

As a result, the items  $\Theta_{55}$  and  $\Theta_{66}$  in the conditions (11), Theorem 2 are replaced by  $-2\varepsilon X + \varepsilon^2\tilde{\mathcal{R}}$  and  $-2\varepsilon\bar{X} + \varepsilon^2\tilde{\mathcal{R}}$ , respectively. Obviously, the resulting conditions with this replacement cover those in Theorem 2.

**Remark 4** Obviously, the solvability of LMI (11) depends on not only the bound of time-delay, but also the actuator fault distribution. Therefore, it will lead to less conservatism in deriving the results.

### 4 An illustrative example

To verify the effectiveness of the proposed method, we consider the following time-delay system (1) with param-

eters:

$$A = \begin{bmatrix} 1 & 2 \\ 1 & -3 \end{bmatrix}, A_d = \begin{bmatrix} -1 & 0.5 \\ -0.5 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$0.01 < \tau(t) < 1.2,$$

and the initial conditions

$$\begin{aligned} \phi(t) &= [-1 \ 1]^T, \\ t &\in [-1.2 \ -0.01]. \end{aligned}$$

Two cases are considered to illustrate the effectiveness of our proposed method. The first one is the case that the systems is in good condition, i.e., there are no any failures in the system. The other one is under the stochastic actuator failure. As shown in Table 1, the standard controller  $K_1$  and reliable controller  $K_2$  are obtained, respectively, according to Theorem 2.

Table 1 Controller.

Case	Fault distribution	Controller
Case 1	$\bar{\Xi}_1 = \begin{bmatrix} 1.0 & 0 \\ 0 & 1.0 \end{bmatrix}, \Delta_1 = \begin{bmatrix} 0.0 & 0 \\ 0 & 0.0 \end{bmatrix}$	$K_1 = \begin{bmatrix} -1.4080 & -2.3273 \\ -0.8026 & 1.4754 \end{bmatrix}$
Case 2	$\bar{\Xi}_2 = \begin{bmatrix} 1.5 & 0 \\ 0 & 1.5 \end{bmatrix}, \Delta_2 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$	$K_2 = \begin{bmatrix} -0.9773 & -1.5832 \\ -0.5386 & 0.8215 \end{bmatrix}$

As shown in Figs. 1–4, when the system is in good condition, listed in Case 1, regardless of using  $K_1$  or  $K_2$ , the system can work well. However, when the system is in failure condition listed in Case 2, the closed-loop system with the standard controller is not even asymptotically stable; while using the reliable controller  $K_2$  can still operate well and maintain an acceptable level of performance.

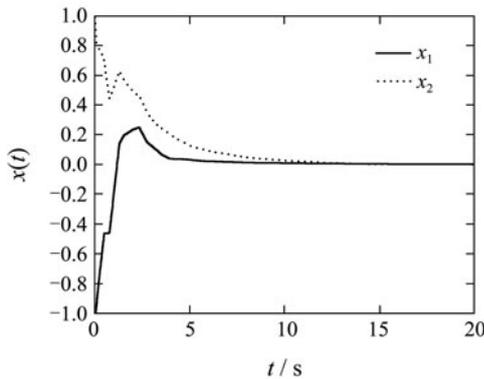


Fig. 1 Standard controller for system (1) without failure.

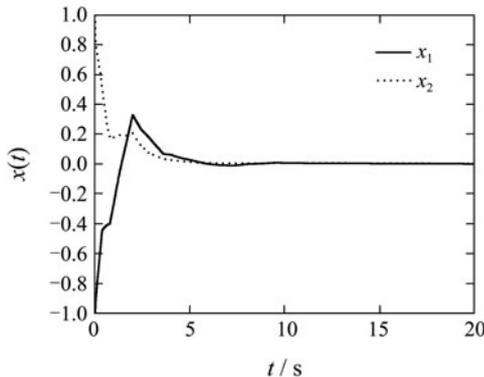


Fig. 2 Reliable controller for system (1) without failure.

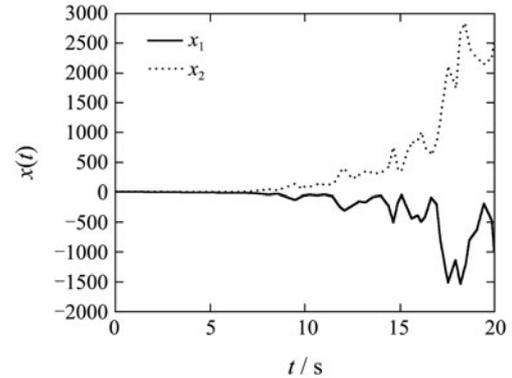


Fig. 3 Standard controller for system (1) with failure.

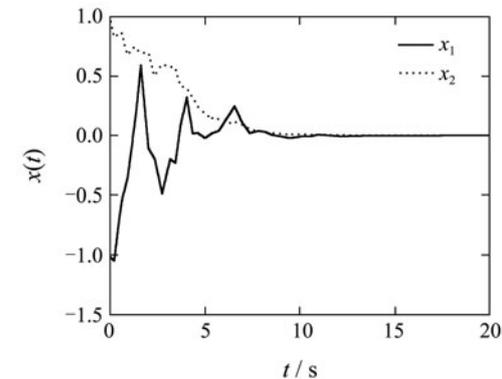


Fig. 4 Reliable controller for system (1) with failure.

### 5 Conclusions

This paper provides a new practical actuator fault model. Based on this, the reliable controller design methodology for continuous-time system with interval time-varying delay is proposed. The system under actuator failure can oper-

ate well by using the proposed reliable controller. Though a numerical example, we illustrate the design procedures.

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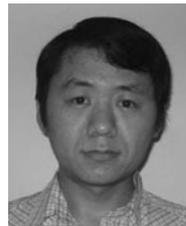
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